2.5 Conservation of Momentum

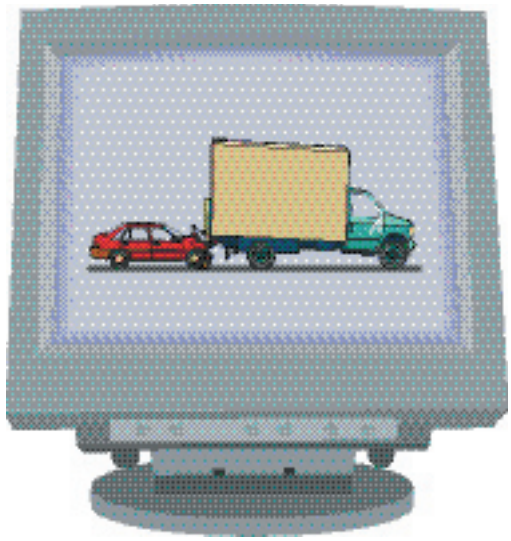
Figure B2.14: Cars are designed with crumple zones to protect the occupants.

The clearest memory that the driver has from the moment the collision occurred was the unmistakable sound of the crushing metal and twisting steel. The impact of the car with the back end of the larger truck did not do much damage to the truck's bumper, but the forces of the collision shortened the total length of the car by about 20 cm.

Although nobody was hurt, at issue was the speed of the car prior to the impact. The accident occurred in a part of the mall parking lot designated as a loading zone for passengers and deliveries into the mall. The posted speed limit is 25 km/h for this area. The driver claims he was only travelling 10 km/h prior to the collision; but the damage to the vehicle leads the insurance company to suspect that the driver was likely travelling much faster. How can such conflicts be resolved? Is there a way to determine the likely speed of the car prior to the collision?

Forensic Engineering

Forensic engineering is a special branch of engineering that deals with answering questions like those posed in the lesson introduction. The investigating engineer collects data from photographs of the scene and performs a careful inspection of the damaged vehicle. Measurements are made of the damage to the car as well as the distance the car and truck rolled forward after the collision. In some cases, state-of-the-art accident-reconstruction software is used to try and piece together the speed of the car prior to the accident.



Momentum is an important consideration when investigating motor vehicle collisions. Investigators frequently use software or perform calculations to determine the total momentum of the vehicles before the collision and the total momentum of the vehicles after the collision. These calculations help determine the likely velocity of each vehicle prior to the collision.

In the next activity you will examine the total momentum before and after collisions using a series of applets called "Momentum and Collisions." Each applet simulates collisions between toy vehicles. You will have the opportunity to change data and collect data for momentum in each applet. You should notice patterns in the momentum data while working with the applets. These patterns will provide insight into how forensic engineers are able to determine the likely velocities of vehicles prior to collisions.

Utilizing Technology

Momentum and Collisions

Purpose

You will collect momentum data before and after a collision or interaction using a series of applets called "Momentum and Collisions" on the Science 20 Textbook CD.

Science Skills

- ✓ Performing and Recording
- ✓ Analyzing and Interpreting

Background Information

This applet shows a number of toy vehicles that can collide in an environment in which external forces, such as friction, are negligible. There are three categories of collisions presented in these applets:

- hit and stick: two vehicles collide, lock together, and move away as one
- hit and rebound: two vehicles collide and move away separately
- explosion: two motionless vehicles, connected by a spring, move in opposite directions once the spring is released

The masses of the vehicles and their velocities prior to the interaction are variables that can be manipulated by the user. Before starting this activity, familiarize yourself with the features of each of these applets by entering values and clicking the Go button. Remember to click the Reset button before entering values for a new trial.

Part A: Hit-and-Stick Collisions

Procedure

step 1: Copy the following data sheet into your notebook or into a spreadsheet program.

Part A: Hit-and-Stick Collisions

Constant Values

Mass of Vehicle 1 = _____

Mass of Vehicle 2 = _____

Velocity of Vehicle 2 Before = _____

Before Collision				After Collision		
Velocity of Vehicle 1 $\vec{v}_1$ (m/s)	Momentum of Vehicle 1 $\vec{p}_1$ (kg·m/s)	Momentum of Vehicle 2 $\vec{p}_2$ (kg·m/s)	Total Momentum $\vec{p}_t$ (kg·m/s)	Mass of Vehicle 1 and Vehicle 2 $m_1 + m_2$ (kg)	Velocity of Vehicle 1 and Vehicle 2 $\vec{v}'_{1 \text{ and } 2}$ (m/s)	Momentum of Vehicle 1 and Vehicle 2 $\vec{p}'_{1 \text{ and } 2}$ (kg·m/s)
+1.00						
+2.00						
+3.00						
+5.00						
+8.00						

Note: The prime symbol (′) is used to denote the value of a quantity after a collision or interaction.

step 2: Select the following constant values: Mass of Vehicle 1 = 2.00 kg, Mass of Vehicle 2 = 3.00 kg, and Velocity of Vehicle 2 Before = -2.00 m/s. Record these values in the appropriate places on the data sheet. They will be kept constant throughout this part of the activity.

step 3: Set the initial velocity of Vehicle 1 to +1.00 m/s. Record the momentum of each vehicle before the collision in the appropriate place on the data sheet.

step 4: Calculate the total momentum before the collision. Record this value on your data sheet.

step 5: Click the Go button, and observe the motion of the vehicles. Record the values of the combined mass, velocity, and momentum for both vehicles after the collision for this trial.

step 6: Repeat steps 3 to 5 for Vehicle 1 velocities of +2.00 m/s, +3.00 m/s, +5.00 m/s, and +8.00 m/s.

Analysis

1. Calculate the total momentum after the collision to confirm the value generated in each trial.
2. For each trial, compare the total momentum before the collision with the total momentum after the collision. Describe the pattern within each pair of values.
3. Explain how adding freight cars onto a train could be another example of a hit-and-stick collision.

Part B: Hit-and-Rebound Collisions

Procedure

step 1: Copy the following data sheet into your notebook or into a spreadsheet.

Part B: Hit-and-Rebound Collisions

Constant Values

Mass of Vehicle 1 = _____

Mass of Vehicle 2 = _____

Velocity of Vehicle 2 Before = _____

Before Collision				After Collision				
Velocity of Vehicle 1 $\vec{v}_1$ (m/s)	Momentum of Vehicle 1 $\vec{p}_1$ (kg·m/s)	Momentum of Vehicle 2 $\vec{p}_2$ (kg·m/s)	Total Momentum $\vec{p}_I$ (kg·m/s)	Velocity of Vehicle 1 $\vec{v}_1$ (m/s)	Velocity of Vehicle 2 $\vec{v}_2$ (kg·m/s)	Momentum of Vehicle 1 $\vec{p}_1$ (kg·m/s)	Momentum of Vehicle 2 $\vec{p}_2$ (kg·m/s)	Total Momentum $\vec{p}_I$ (kg·m/s)
+1.00								
+2.00								
+4.00								
+8.00								

step 2: Select the following constant values: Mass of Vehicle 1 = 2.00 kg, Mass of Vehicle 2 = 8.00 kg, and Velocity of Vehicle 2 Before = -1.00 m/s. Record these values in the appropriate places on the data sheet. They will be kept constant throughout this part of the activity.

step 3: Set the initial velocity of Vehicle 1 to +1.00 m/s. Record the momentum of each vehicle before the collision in the appropriate place on the data sheet.

step 4: Calculate the total momentum before the collision. Record this value on your data sheet.

step 5: Click the Go button, and observe the motion of the vehicles. Record the values of the velocity and the momentum for each vehicle after the collision.

step 6: Repeat steps 3 to 5 for Vehicle 1 velocities of +2.00 m/s, +4.00 m/s, and +8.00 m/s.

Analysis

4. Calculate the total momentum after the collision to confirm the value generated in each trial.
5. For each trial, compare the total momentum before the collision with the total momentum after the collision. Describe the pattern within each pair of values.
6. Explain how two billiard balls colliding could be another example of a hit-and-rebound collision.

Part C: Explosions

In this part, the two toy cars are connected by a spring. The two vehicles sit motionless until the spring uncoils.

Procedure

step 1: Copy the following data sheet into your notebook or into a spreadsheet.

Part C: Explosions

Constant Values

Velocity of Vehicle 1 Before = _____

Velocity of Vehicle 2 Before = _____

		Before Collision			After Collision				
Mass of Vehicle 1 m_1 (kg)	Mass of Vehicle 2 m_2 (kg)	Momentum of Vehicle 1 $\vec{p}_1$ (kg·m/s)	Momentum of Vehicle 2 $\vec{p}_2$ (kg·m/s)	Total Momentum $\vec{p}_t$ (kg·m/s)	Velocity of Vehicle 1 $\vec{v}_1$ (m/s)	Velocity of Vehicle 2 $\vec{v}_2$ (m/s)	Momentum of Vehicle 1 $\vec{p}'_1$ (kg·m/s)	Momentum of Vehicle 2 $\vec{p}'_2$ (kg·m/s)	Total Momentum $\vec{p}'_t$ (kg·m/s)
1.00	8.00								
2.00	5.00								
4.00	4.00								
5.00	4.00								

step 2: Because this simulates an explosion of an object initially at rest, record the velocity of Vehicles 1 and 2 before the interaction as being zero on the data sheet.

step 3: Set the initial value of the mass of Vehicle 1 to 1.00 kg and the mass of Vehicle 2 to 8.00 kg.

step 4: Calculate the momentum of each vehicle as well as the total momentum before the explosion. Record these values on your data sheet.

step 5: Click the Go button, and observe the motion of the vehicles. Record the values of the velocity and momentum for each vehicle after the explosion.

step 6: Repeat steps 3 to 5 for the other pairs of mass values shown on the data sheet.

Analysis

7. Calculate the total momentum after the explosion to confirm the value generated in each trial.
8. For each trial, compare the total momentum before the collision with the total momentum after the collision. Describe the pattern within each pair of values.
9. Explain how a bullet leaving the barrel of a rifle could be another example of an explosion interaction.

Evaluation

10. You studied three types of interactions: hit and stick, hit and rebound, and explosion. Identify the overall trend within the pairs of values before and after an interaction.
11. The applets simulated interactions between toy cars in a frictionless environment. If the force of friction played a significant role, how would the motion of the vehicles have differed?

Momentum Is Conserved

The collisions that occurred in the activity are considered to be one dimensional because the collisions of the objects occurred in a straight line. Further, these collisions can be classified into three basic types of collisions: hit and rebound, hit and stick, and explosion.

Regardless of the type of collision or interaction, it can be concluded that momentum can be transferred from one object to another. In all cases, the momentum gained by one object equals the momentum lost by the other so that the total amount of momentum remains constant. This is called the **law of conservation of momentum**. This law can be stated in equation form by having the Greek letter sigma, Â, represent the phrase "the sum of."

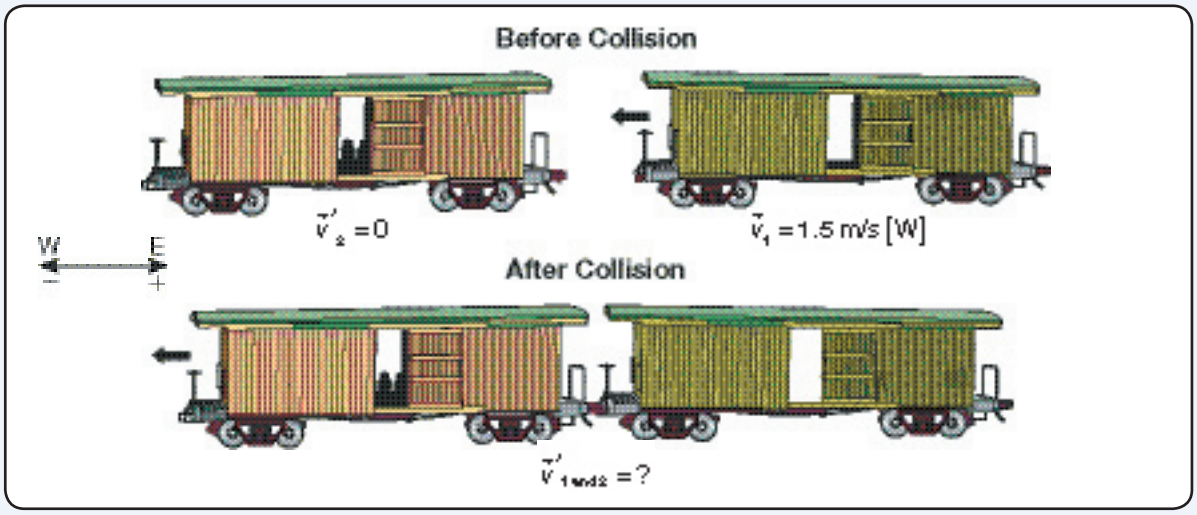
$$\sum \vec{p}_{before} = \sum \vec{p}_{after}$$

This equation is read as, "The sum of the momentum before the interaction is equal to the sum of the momentum after the interaction." When problem solving, it is more convenient to substitute the subscripts "before" and "after" by using the prime symbol (′) to indicate quantities that occurred after the interaction. The following example problems illustrate how to apply the law of conservation of momentum.

law of conservation of momentum: a law stating that if the net force acting on a system is zero, the sum of the momentum before an interaction equals the sum of the momentum after the interaction

Example Problem 2.10

A 10 000-kg freight car travelling west with a velocity of 1.5 m/s collides with a 20 000-kg freight car at rest. After the collision, the freight cars stick together. Determine the velocity of the freight cars after the collision.



Solution

Let east be the positive direction.

$m_1 = 10\,000$ kg

$m_2 = 20\,000$ kg

$\vec{v}_1 = 1.5$ m/s [W]
$= -1.5$ m/s

$\vec{v}_2 = 0$

$\vec{v}'_{1 \text{ and } 2} = ?$

Note: Because the freight cars lock together after the collision, the cars can be treated as one object (1 and 2).

$$\sum \vec{p}_{before} = \sum \vec{p}_{after}$$

$$\vec{p}_1 + \vec{p}_2 = \vec{p}'_{1 \text{ and } 2}$$

$$m_1\vec{v}_1 + m_2\vec{v}_2 = (m_1 + m_2)\vec{v}'_{1 \text{ and } 2}$$

$$m_1\vec{v}_1 = (m_1 + m_2)\vec{v}'_{1 \text{ and } 2} \quad \leftarrow \vec{v}_2 = 0$$

$$(10\,000 \text{ kg})(-1.5 \text{ m/s}) = (10000 \text{ kg} + 20000 \text{ kg})\vec{v}'_{1 \text{ and } 2}$$

$$-1.5 \times 10^4 \text{ kg·m/s} = (30000 \text{ kg})\vec{v}'_{1 \text{ and } 2}$$

$$(30\,000 \text{ kg})\vec{v}'_{1 \text{ and } 2} = -1.5 \times 10^4 \text{ kg·m/s}$$

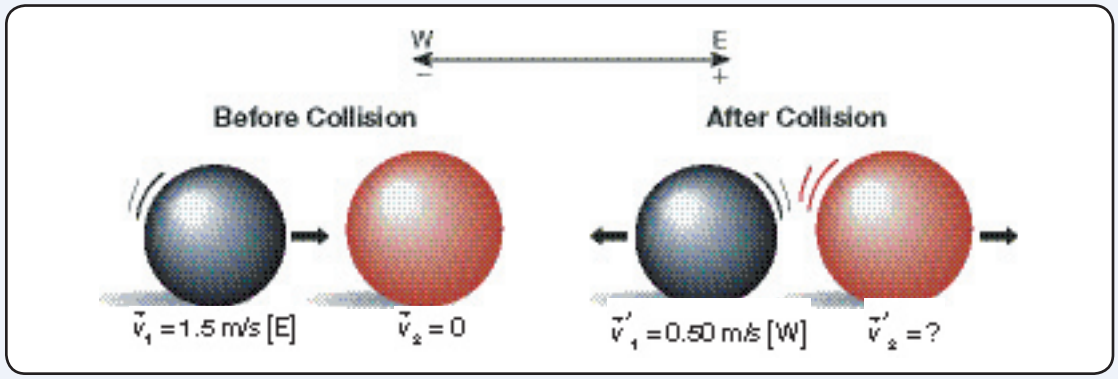
$$\vec{v}'_{1 \text{ and } 2} = \frac{-1.5 \times 10^4 \text{ kg·m/s}}{30\,000 \text{ kg}}$$

$$= -0.50 \text{ m/s}$$

The freight cars have a velocity of 0.50 m/s[W] after the collision.

Example Problem 2.11

A 3.0-kg ball rolling east with a velocity of 1.5 m/s collides with another 6.0-kg ball at rest. After the collision, the first ball rebounds and is travelling at a velocity of 0.50 m/s[W].



a. Determine the velocity of the second ball after the collision.
b. Determine the momentum values of the balls before and after the collision.
c. Use scale diagrams of momentum vectors to demonstrate that momentum is conserved in this collision.

Solution

Let east be the positive direction.

a. $m_1 = 3.0$ kg

$m_2 = 6.0$ kg

$\vec{v}_1 = 1.5$ m/s [E]
$= +1.5$ m/s

$\vec{v}_2 = 0$

$\vec{v}'_1 = 0.50$ m/s [W]
$= -0.50$ m/s

$\vec{v}'_2 = ?$

$$\sum \vec{p}_{before} = \sum \vec{p}_{after}$$
$$\vec{p}_1 + \vec{p}_2 = \vec{p}'_1 + \vec{p}'_2$$
$$m_1\vec{v}_1 + m_2\vec{v}_2 = m_1\vec{v}'_1 + m_2\vec{v}'_2$$
$$m_1\vec{v}_1 = m_1\vec{v}'_1 + m_2\vec{v}'_2 \quad \leftarrow \vec{v}_2 = 0$$
$$(3.0\text{ kg})(+1.5\text{ m/s}) = (3.0\text{ kg})(-0.50\text{ m/s}) + (6.0\text{ kg})\vec{v}'_2$$
$$+4.5\text{ kg·m/s} = -1.50\text{ kg·m/s} + (6.0\text{ kg})\vec{v}'_2$$
$$(6.0\text{ kg})\vec{v}'_2 = (+4.5\text{ kg·m/s}) - (-1.50\text{ kg·m/s})$$
$$\vec{v}'_2 = \frac{(+4.5\text{ kg·m/s}) - (-1.50\text{ kg·m/s})}{6.0\text{ kg}}$$
$$= +1.0\text{ m/s}$$

The velocity of the second ball is 1.0 m/s[E].

b. **Before Collision**

$\vec{p}_1 = m_1\vec{v}_1$
$= (3.0\text{ kg})(+1.5\text{ m/s})$
$= +4.5\text{ kg·m/s}$

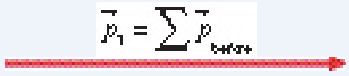
After Collision

$\vec{p}'_1 = m_1\vec{v}'_1$
$= (3.0\text{ kg})(-0.50\text{ m/s})$
$= -1.5\text{ kg·m/s}$

$\vec{p}'_2 = m_2\vec{v}'_2$
$= (6.0\text{ kg})(+1.0\text{ m/s})$
$= +6.0\text{ kg·m/s}$

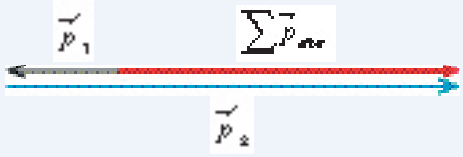
c. scale: 1 cm = 1 kg·m/s

Before Collision



The sum of the momentum before the collision is +4.5 kg·m/s.

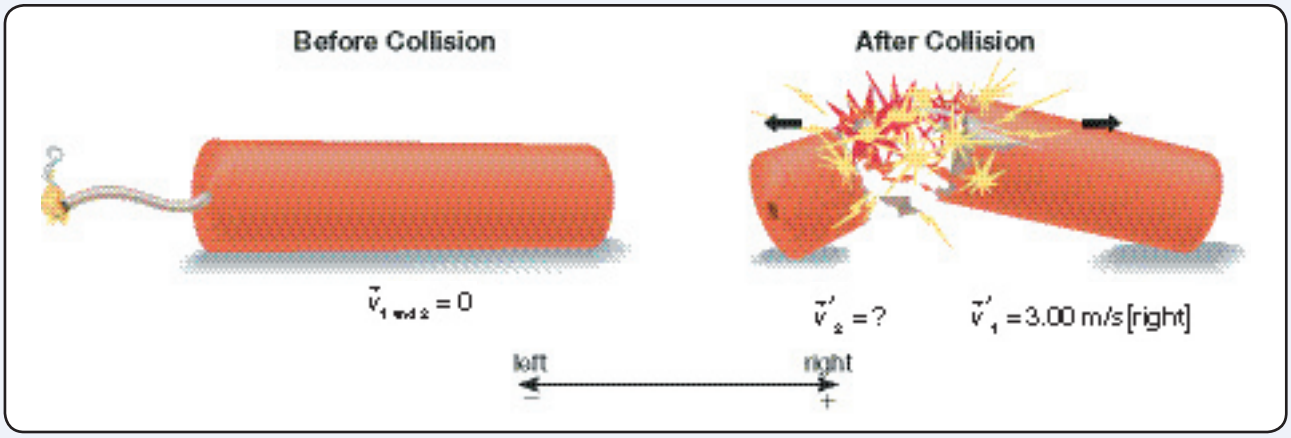
After Collision



The sum of the momentum after the collision is +4.5 kg·m/s.

Example Problem 2.12

A 0.020-kg firecracker at rest explodes into two pieces. If a 0.015-kg piece flies off to the right at a velocity of 3.00 m/s, determine the velocity of the other 0.0050-kg piece.



Solution

Let right be the positive direction.

$m_{1 \text{ and } 2} = 0.020 \text{ kg}$

$\vec{v}_{1 \text{ and } 2} = 0$

$m_1 = 0.015 \text{ kg}$

$m_2 = 0.0050 \text{ kg}$

$\vec{v}'_1 = 3.00 \text{ m/s[right]}$
$= +3.00 \text{ m/s}$

$\vec{v}'_2 = ?$

$$\sum \vec{p}_{\text{before}} = \sum \vec{p}_{\text{after}}$$
$$\vec{p}_{1 \text{ and } 2} = \vec{p}'_1 + \vec{p}'_2$$
$$m_{1 \text{ and } 2}\vec{v}_{1 \text{ and } 2} = m_1\vec{v}'_1 + m_2\vec{v}'_2$$
$$(0.020 \text{ kg})(0) = (0.015 \text{ kg})(+3.00 \text{ m/s}) + (0.0050 \text{ kg})\vec{v}'_2$$
$$0 = (+0.045 \text{ kg·m/s}) + (0.0050 \text{ kg})\vec{v}'_2$$
$$(0.0050 \text{ kg})\vec{v}'_2 = -0.045 \text{ kg·m/s}$$
$$\vec{v}'_2 = \frac{-0.045 \text{ kg·m/s}}{0.0050 \text{ kg}}$$
$$= -9.0 \text{ m/s}$$

The 0.0050-kg piece of the firecracker has a velocity of 9.0 m/s[left].

Practice

17. A 0.010-kg marble rolling on the floor with a velocity of 0.90 m/s[E] collides with a 0.015-kg marble at rest. The first marble rebounds with a velocity of 0.30 m/s[W].
    a. Determine the velocity of the second marble.
    b. Determine the momentum of the marbles before and after the collision.
    c. Use scale diagrams, such as the head-to-tail method, to demonstrate that momentum is conserved in this collision.
18. A 0.010-kg marble rolling on the floor with a velocity of 0.90 m/s[N] collides with a 0.015-kg ball of putty at rest. If the two stick together, determine their combined velocity immediately after the collision.
19. A 0.0100-kg balloon, initially at rest, explodes into two pieces. If a 0.0090-kg piece is propelled with a velocity of 4.0 m/s[N], determine the velocity of the other 0.0010-kg piece.

Applications to Forensic Engineering

The story at the beginning of this lesson described a collision between a small car and a stationary truck. In this scenario the driver claimed to be only travelling at 10 km/h, but the investigators for the insurance company suspected that the car was travelling much faster. Now that you know about the law of conservation of momentum, you can solve the mystery in Practice question 20.

Practice

20. A collision occurred when a 1500-kg car collided with a 15 000-kg truck initially at rest. Since neither vehicle had its brakes engaged, the external forces present can be considered minimal. After the collision, the vehicles locked together and rolled with a combined velocity of 1.0 m/s, east.
    a. What type of collision is described by this situation?
    b. Use the law of conservation of momentum to solve for the velocity of the car, in metres per second, before the collision occurred.
    c. Convert your answer to question 20.b. into kilometres per hour.
    d. The posted speed limit in this area was 25 km/h, because it was designated for loading passengers and freight. Was the car speeding?

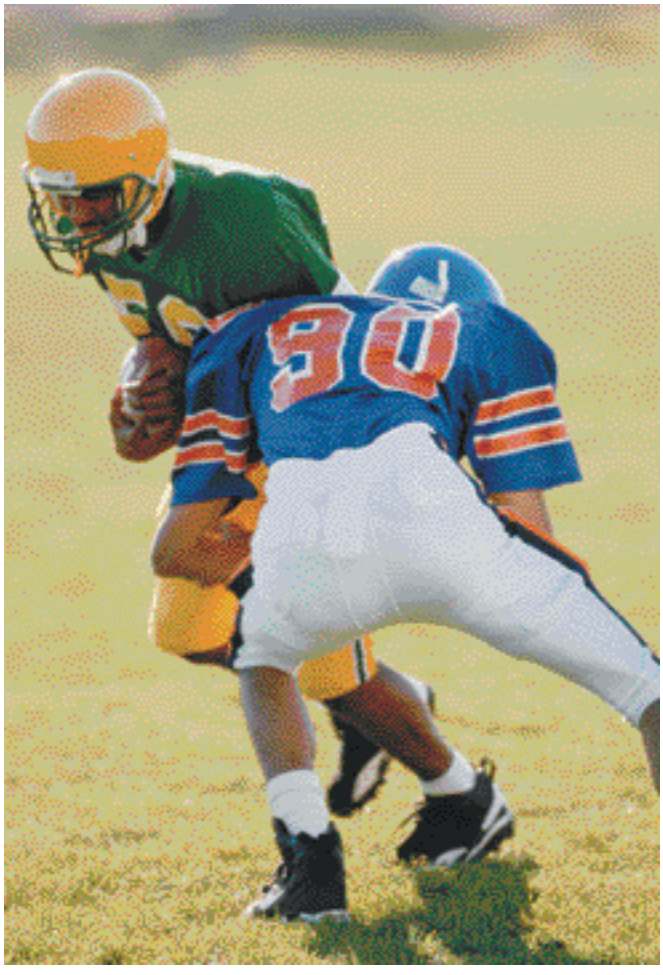
2.5 Summary

In this lesson you used applets to investigate the role of momentum in a number of one-dimensional interactions: hit and stick, hit and rebound, and explosion. In all cases, if the net force acting on a system is zero, the sum of the momentum before an interaction equals the sum of the momentum after the interaction. This is called the law of conservation of momentum. The law can be applied to one-dimensional problems, including problems that deal with the analysis of collision data.

2.5 Questions

Knowledge

1. Describe what is meant by a one-dimensional collision.
2. Name the three types of collisions.
3. Describe the law of conservation of momentum in word and equation form.

Applying Concepts

4. A 3500-kg truck travelling 20.0 m/s[E] strikes a 2000-kg, parked minivan. After the collision, the parked minivan is propelled forward with a velocity of 14.0 m/s[E]. Determine the final velocity of the truck.
5. A 3500-kg truck travelling 20.0 m/s[E] strikes a 2000-kg, parked minivan. After the collision, the truck sticks to the minivan. Determine the combined final velocity of the truck and the minivan.
6. A 2500-kg van travelling 80.0 km/h[E] strikes a 1500-kg compact car at rest. After the collision, the 1500-kg car propels forward with a velocity of 60.0 km/h[E]. Determine the final velocity of the van in kilometres per hour.
7. A 20.0-g inflated balloon, initially at rest, is punctured with a nail and explodes into two pieces. If a 12.0-g piece of balloon explodes to the left with a velocity of 3.00 m/s, calculate the final velocity of the other 8.0-g piece.
8. A 75-kg football player running 1.50 m/s[E] is tackled by another player who is initially at rest. After the collision, the football players stick together and travel with a combined velocity of 0.80 m/s[E]. Calculate the mass of the other player.